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COST GROWTH: EFFECTS OF SHARE RATIO AND RANGE OF INCENTIVE EFFECTIVENESS

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13. ABSTRACT

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The important major findings are that:

a. The use of the most probable cost for target costs (directed by ASPR) as opposed to expected cost, produces about 20 percent contract cost growth.

b. There is a positive correlation between contractor's share of underrun and contractual adjustments and a negative correlation between overrun and the contractor's share for overruns.

c. The contractor's share of underrun and overrun is less than the negotiated share, on the average, while his profit for final costs which are above the upper limit of the range of incentive effectiveness is occasionally far greater than the negotiated minimum profit.

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by

Robert L. Launer, Ph.D.

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U.S. Army Procurement Research Office
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SUMMARY

1. Background and Purposes.

In previous cost growth research performed in this office, the CPIF type of contract was found to be associated more frequently and in greater intensity with contract cost growth than any other contract type. Further analysis of this research finding suggested some potentially useful insights regarding relationships between cost growth on the one hand and share ratios, target costs, overruns/underruns, and the ranges of incentive effectiveness on the other hand. The purpose of this study is to provide a closer examination of these relationships.

2. Nature and Scope of Study.

The study is analytic in nature and many sources have been utilized in generating data including forms DD 1500, DD 350, Contractor Performance Evaluation Reports (CPE), and the contract file data extracted from a random sample of 300 AMC procurements. These data were used to construct the cost growth profile of contract types, type of work, commodities, and many other categories.

3. Findings.

The findings of this report are listed below.

a. The negotiated share ratio and the estimated share ratio (as estimated from the final cost and profit) are not correlated. This information is important to anyone conducting studies involving CPIF contracts.

b. The use of "most probable costs" (directed by ASPR) as opposed to the use of expected costs in negotiation procedures induces about 20 percent contract cost growth.

c. There is a positive correlation between the dollar cost of contractual adjustments and the contractor's share of the underrun, and there is a negative correlation between the overrun and the contractor's share for overrun. This indicates that the slope of the "share line" affects overruns and underruns in the intended way. It was also found that the cost of contractual adjustments as a percent of the contract's initial cost is highly correlated with the difference between the contractor's share of overrun. These collective results constitute a partial confirmation of the existence of the contractor buy-in.

d. The share of both the underrun and the overrun which the contractor actually receives is, on the average, less than the negotiated share within and above the range of incentive effectiveness. In other words, the contractor is receiving more profit for overruns and less profit for underruns than that indicated by the share line.

4. Recommendations.

The results of this study are of such a nature, that clear-cut policy recommendations would be exceedingly difficult to make. Therefore, the recommendations are pointed at either senior Army policy makers for consideration in policy formulation, or else for analysts and researchers who offer advice to the policy makers or who conduct investigations in logistics problems.

a. It is recommended that senior DA procurement analysts be made aware of the relationship between the mean and the mode in CPIF cost data. The obvious but simplistic recommendation that "expected" costs should be used instead of "most probable" costs will be avoided here, because of the possible effect of the resulting higher target costs on the final costs. Any further policy recommendation will require more study tempered with sound procurement judgment.

b. It is recommended that procurement analysts be made aware of the disparity between the contractors negotiated share of overrun and underrun and the (smaller) share which he receives on the average. A more extensive recommendation will not be given since this disparity works to the advantage of the Government in the underrun situation. There is also the possibility of the existence of hidden "trade-offs" between the Government and the contractor, which are not measurable from the data, but which work to the benefit of the Government.

c. It is recommended that studies performed within the Department ot the Army involving the affect of the share ratio in CPIF contracts on cost growth should use the negotiated share ratio obtained from the contract files. Analysis which is based on share ratios estimated from the final cost data may be highly inaccurate.

CHAPTER I

INTRODUCTION

A. Purpose.

The general purpose of this study is a close examination of several problem areas and relationships related to CPIF contracts. In particular, this study is aimed at investigating the relationship between cost growth patterns and (a) the magnitude of the share ratio; (b) target costs derived from estimated most probable costs, as opposed to estimated expected costs; (c) the difference in the share ratio between underrun and overrun; and (d) the range of incentive effectiveness along with the contractor's management of contract modifications.

Potentially useful insights regarding these relationships were observed in previous cost growth research, [2, 4, 7, 8, 10, 15]. For example, the following statement was made in the immediately preceding cost growth report [8]:

> "Of the net 6.9 percent cost growth attributable to cost overrun, 22 overruns increased cost growth by 9.0 percent, while the 28 underruns decreased cost growth by only 2.1 percent. Overruns and underruns were fairly evenly distributed within the cost range in which cost incentive features of the contract were effective. Once the cost exceeded the upper range of incentive effectiveness however, large overruns were recorded. Moreover, no instances were observed where the actual costs fell below that level where the contractor earned maximum allowable fee. Thus, it appears that contractors made no attempt to control costs when the cost was significantly outside the incentive range. Additional analysis is being performed in this area."

This statement prompted the study of (d) above.

B. Scope and Method.

The study is primarily a statistical analysis of 53 randomly selected CPIF contracts performed for the Department of the Army between 1964 and 1971. All contracts in the sample were definitized at $500,000 or more. The procurements studied include contracts with contractors share ratios ranging from 9.36 percent to 50 percent. The relationships between cost estimates, final cost, share ratios, and ranges of incentive effectiveness are studied by means of correlation and regression analysis, "piecewise linear regression," analysis of variance and the estimation of density functions and modes. The appropriateness of each of these quantitative methods to the particular relationship being studied is explained along with the analysis in the text. Of special note, the "piecewise linear regression" technique was adopted especially for this study to examine the effect of the range of incentive effectiveness because the share line is piecewise linear.

C. Desired Objectives.

The desired objectives of the study at the outset were to determine.

1. Whether or not there is a tendency for contractors to attempt to control costs only so long as the cost is within the range of incentive effectiveness.

2. That if there is such a tendency (implying that the contractor has a great deal of control over the final cost), is it possible to detect this control, perhaps in the form of a buy-in?

3. What ultimate effect the practice of using most probable costs for target costs has on cost growth.

4. Whether or not the magnitude of the contractor's share is related to cost growth. If no relationship is detected, is this a result of both the share and cost growth being influenced by the uncertainty in the contract?

5. Whether or not the degree of break in the share between overrun and underrun is related to cost growth. If no relationship is detected, is this a result of both the break in the share line and cost growth being influenced by the uncertainty in the contract?

6. Whether the range of incentive effectiveness should be extended or not.

7. Whether the correlation between the negotiated share ratio and the share ratio as "estimated" from the contract final cost and profit figures is high enough to warrant using this estimated share ratio in analyses which are based on negotiated share ratios.

D. Description of Incentive Structure.

The structure of the incentive share ratio (S/R) and the range of incentive effectiveness (RIE) for cost and multiple incentive contracts is discussed thoroughly in the "Incentive Contracting Guide" [4]. Briefly, the share ratio reflects the percent of the difference between target cost and final cost which is given to the contractor if an underrun is experienced, or taken from the contractor in the case of an overrun. In order to make this system reasonable, a minimum profit and a maximum profit are imposed and the costs which correspond to maximum and minimum profit are referred to as the lower and upper limits of the RIE, respectively. The RIE is, therefore, that range of values of the cost over which the S/R operates. The share ratio is expressed as a ratio of the form X/(100-X), where X is the percent of the overrun (underrun) which the Government pays (keeps). By law, the maximum profit that may be earned in a CPIF contract is 15 percent of the target cost for R&D contracts and 10 percent of the target cost for others (ASPR 3-405.6). Thus, the limits of the RIE are usually specified in the contract by the maximum and minimum profit, expressed as a percent of target cost.

It frequently happens that a "broken" S/R is negotiated for a particular CPIF contract. That is, the share ratio for an overrun (O/R) may be different from the S/R for an underrun (U/R). In fact, it also happens, that the S/R may vary for different values of the overrun (or U/R). For examples, the S/R could be 90/10 if the O/R is up to $1,000,000; 85/15 if the O/R is between $1M and $2M and 80/20 if O/R is over $2M, but less than the maximum limit prescribed by the terms of the contract.

The exact value(s) of the S/R is usually determined by negotiation, and theoretically reflects the amount of risk inherent in the work to be performed under the terms of the contract. A contractor who believes that there is a great deal of risk or uncertainty involved in a particular contract will, on the average, not accept an S/R as "large" as the S/R that he would accept on a contract in which there is very little risk. In other words, a contractor may be willing to accept a 30 percent or even a 50 percent share of O/R for a contract to which he attaches little or no uncertainty but would accept no "more" than a 10 percent for a very risky contract.

CHAPTER II

CORRELATION OF NEGOTIATED SHARE RATIO WITH THE COMPUTED SHARE RATIO

The research contained in this study was condu ed to answer several specific questions about the cost growth profile of the CPIF contract type. (See Chapter I.) These questions involve the negotiated share ratio and range of incentive effectiveness either directly or indirectly. Several studies have been written which treat these or similar questions, but they all appear to use estimates of the share ratio instead of obtaining the actual negotiated share ratio. Admittedly, obtaining the negotiated figures is far more costly and time consuming than estimating them. There are, however, several reasons why the estimated share ratio could very reasonably be expected to differ from the negotiated share ratio. One reason is that the range of incentive effectiveness produces a break point in the "share line."

Most studies estimate the share ratio from the final contract cost and profit and then treat it in the analysis as if it were the negotiated share ratio which presumably affects the final cost and profit (and, therefore, affects the estimated share ratio.)

If the estimated share ratio is always in very close agreement with the negotiated share ratio (say within 1 percent or 2 percent) then substituting one for the other would seem to be an acceptable procedure. If the agreement is not close, then the practice of utilizing these estimates in an analysis is open to serious question. It was decided, therefore, to begin the study with a correlation analysis between the estimated and negotiated share ratios. The results of this analysis would determine the need and appropriateness of examining additional relationships, such as those cited in the purpose stated above.

The incentive share ratio is related to the contract costs and profit in the following way.

Let,
Π_A = Adjusted target profit
Π_F = Final profit
C_A = Adjusted target cost
C_F = Final cost
S = Incentive share ratio

Then, $\Pi_A - \Pi_F = S(C_F - C_A)$, and so

$$S = \frac{\Pi_A - \Pi_F}{C_F - C_A} \qquad (1)$$

Since the profit awarded on CPIF contracts is rarely precisely the same as that given by the negotiated share line, formula (1) will not in general agree with the negotiated share ratio. The most likely causes of this disparity are multiple incentives, broken share lines, and the final contract renegotiation and closeout. These are, however, not the only causes.

Figure 1 is a scatter plot of the computed share ratio (from formula (1)) versus the negotiated share ratio. Since the negotiated share ratio for overruns will frequently differ from the underrun negotiated share ratio on a given contract, care was taken to use the appropriate figure in the correlation analysis. (Overruns and underruns were analyzed separately to see if this would improve the resulting estimates). Furthermore, in order to avoid ambiguity, a contract which experienced an overrun and which had a broken share ratio for overruns, was not included, and similarly for underruns.

Table I gives the results of the analysis. The t-values are computed from the formula:

$$t = \frac{r}{\sqrt{1-r^2}}\sqrt{n-2} \sim t_{\alpha/2,\, n-2}$$

where r is the estimated correlation coefficient:

$$r = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}}$$

Notice that the t statistics are all quite low. The largest one fails to be significant at even the .20 level. This indicates that the correlation between the negotiated share ratio and the share ratio as computed after the fact from formula (1) is practically zero.

The conclusion to be drawn from this result is that estimates of share ratios from cost data (such as the form DD 1500) are extremely inaccurate and inferences obtained through their use are invalid. This conclusion is not an indictment of the accuracy of the DD 1500 data, but rather points out the fact that contract cost incentive payments are affected by such things as schedule and performance incentives, broken share ratios, and the range of incentive effectiveness.

IMPUTED SHARE RATIO VS. NEGOTIATED SHARE RATIO
(Does Not Include Broken Share Ratio)
(Notice the Negative Imputed Share Ratio)

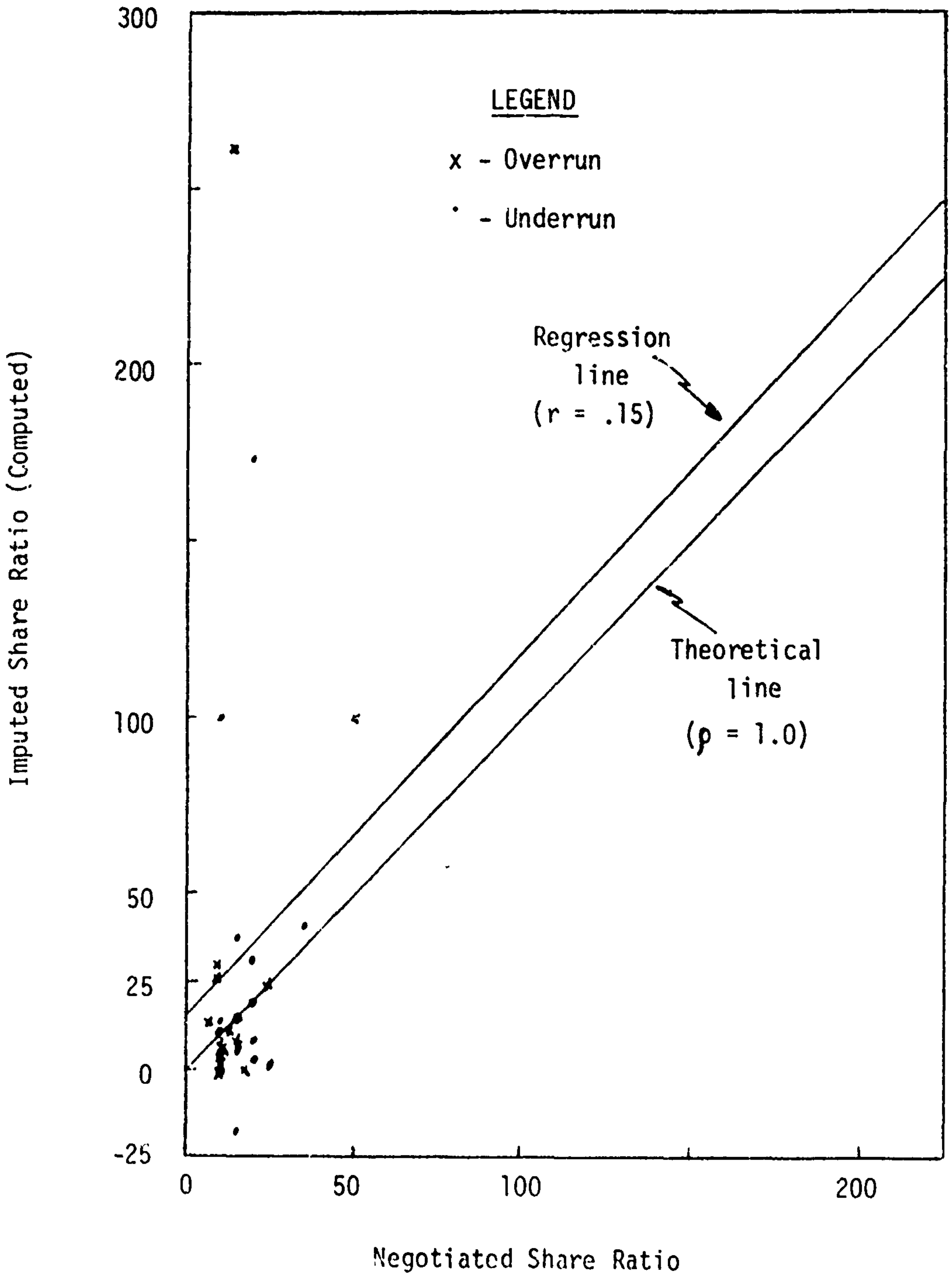


FIGURE 1

TABLE I

CORRELATION ANALYSIS OF NEGOTIATED SHARE RATIO WITH COMPUTED SHARE RATIO

Entire Sample

29 CPIF Contracts

r = .15	b = 1.02	t = .79

$t_{27,.05} = 1.7$

Underruns only

16 CPIF Contracts

r = .02	b = .18	t = .07

$t_{14,.05} = 1.76$

Overruns Only

12 CPIF Contracts

r = .23	b = 1.5	t = .75

$t_{10,.05} = 1.8$

CHAPTER III

THE ROLE OF COST ESTIMATES IN THE COST GROWTH OF CPIF CONTRACTS

A. Introduction.

There are many established as well as potential but not necessarily positively identified producers of cost growth. One cause which is often mentioned but rarely, if ever, carefully analyzed for its effect on cost growth is the cost estimate itself. The reason for this is not lack of interest but is due to the lack of appropriate data. This is a result of the fact that the notion of a "true" contract cost (in the absolute sense) is so nebulous.

If it were possible for several contractors to work simultaneously and independently on identical contracts, with no changes in scope, quantity, engineering, or schedule issued by the Government, and with no price competition among the contractors, then there would probably be several different cost estimates for the various contractors and several different final costs for each of the contracts. Furthermore, the cost estimates and final cost experienced by a given contractor would almost certainly differ!

It seems clear from the preceding paragraph, that a direct comparison of a cost estimate with its "true" cost is not possible. This suggests that an indirect analysis based on statistical methods might be advantageous. Therefore, the cost estimate and the negotiated cost of a contract will be treated as if they were random variables. (The word random in this context does not mean haphazard but, rather, unpredictable.) A random variable (r.v.) is usually characterized by descriptive parameters such as "expected value" (overall average) and "variance" (spread or dispersion). The true cost mentioned previously, corresponds most closely with the concept of expected value.

The "Armed Services Procurement Regulation" (ASPR) and the "Incentive Contracting Guide" are written with this in mind.

The following three quotations are taken from the "Incentive Contracting Guide," October 1969, FM 38-34.

> (1) The ingredients of a cost-plus-incentive-fee contract (CPIF) are:
>
> (i) Target Cost (the most probable cost for target performance)
> (ii) Target Fee (a reasonable fee for target performance)
> (iii) Maximum Fee (subject to Agency control)
> (iv) Minimum Fee (may be a "negative fee")
> (v) Share Formula (the arrangement for establishing final fee)

> You will have observed that the definition of target cost above as the most probable cost for target performance is different than the definition given previously for target cost under the fixed-price incentive contract coverage as the cost "against which to measure final costs." For either contract type the latter description of "most probable cost" applies to target cost. (P. 79–80.)
>
> (2) Much discussion centers on the question "What is a 'good' target?" It has been suggested that, "A good target cost is one about which both parties can agree there is an equal chance of either overrunning or underrunning basing their judgment on all complete and current facts available at a point in time. . . .the estimated target cost should be one of equal chance of overrunning or underrunning, not equal magnitude. The idea of symmetry has somehow crept in and people tend to say a target cost is good + or - 20 percent. This is rarely true. The magnitude of the potential overrun usually will not equal the magnitude of the potential underrun. (P. 85.)
>
> (3) The target cost should represent the best, mutually determined estimate of what costs will actually be when incurred, or, stated another way, that target cost should represent that figure at which there is equal probability of either a cost underrun or overrun. (P. 87.)

Several important observations should be made at this point. Quotation (1) refers to a target cost as "the most probable cost." The most probable value of a random variable is called its MODE [5]. Quotation (2) says that a good target cost is one for which the probability of an overrun or underrun are equal. This value of a random variable is its MEDIAN, [5]. Finally, quotation (3) says that the target cost should represent the best estimate of what costs will actually be. This ordinarily means the expected value or MEAN of a random variable. (The remainder of quotation (3) then says that this "expected cost" is the "median" cost of quotation (2), but since the term, "estimate of what costs will actually be" is vague, nothing more will be said about this last point.)

In general, unless the distribution of a continuous random variable is symmetric, its mean, median, and mode will all differ from one another. The claim is made in quotation (2) that overruns and underruns are not symmetric, which then means that the statements

made in the quotations are contradictory. Since the meaning of the mean, median, and mode are of little interest to the procurement analyst, this discussion at first seems hardly worthy of development here. There is, however, one very important consequence of all of this for cost growth measurements.

One measure of the asymmetry of the distribution of a r.v. is its skewness, τ, which is defined to be the difference of the mean μ and mode divided by the standard deviation, (S.D.) [3].
That is,

$$\tau = \frac{\text{mean - mode}}{\text{S.D.}}$$

If the mean is greater than the mode, then the skewness is positive. One type of distribution which has positive skew is one which has a relatively long tail to the right and is bounded on the left.

Cost growth is customarily estimated from sample averages and the sample average is an estimate of the "mean" cost growth. Since the means of both cost growth and overruns are greater than their respective modes (Section B), then the use of most probable costs as target costs will invariably result in cost increases as computed by average cost growth.

B. The Skewness and Mode of CPIF Overruns and Adjustments.

In an effort to estimate just how much cost growth is generated by "modal" estimation of contract costs, it was decided to estimate these from the sample. The estimation of the mode of a r.v. is relatively new. Parzen [11] has obtained such a procedure. (See appendix I for a brief description.)

Figure 2 is a sketch of the estimate of the distribution of cost growth as a percent of initial cost for the entire sample of 53 contracts. The mean and mode are approximately 57 percent and 37 percent cost growth respectively. The overruns distribution and mode are given in figure 3 with mean and mode .04 and .008 respectively.

Since eight of the contracts did have quantity changes, the procedure was repeated for cost growth less the quantity and for the contractual adjustments less overrun and less quantity changes. In both cases, the mean is about 20 percent greater than the mode, as depicted in figures 4 and 5.

Since quantity changes are apt to produce overruns, it would be useful to examine the distribution of cost growth without the quantity changes and without that part of the overrun which the quantity changes produce. This data is not available, but can be estimated if one is willing to prorate quantity changes so that the ratio of overruns produced by the quantity changes to the actual (recorded) overrun is the same as the ratio of the dollar value of the quantity changes to the dollar value of the initial target cost plus quantity changes. That is to say, if a contract experiences a quantity change which doubles the original

quantity then 50 percent of the overrun is due to the quantity change and the remaining 50 percent is the overrun that "would have been," without any quantity change. That fraction of the overrun which is due only to the quantity ordered in the original contract will be referred to as overrun prorated to initial cost.

Let:

C_I = Initial target cost
C_A = Adjusted target cost
Q = Cost of quantity increase
R = Overrun
R_p = Overrun prorated to C_I

Then,

$$\frac{R_p}{R} = \frac{Q}{C_I + Q},$$

$$R_p = R \cdot \frac{Q}{C_I + Q}.$$

The overrun and quantity changes may be expressed as a percent of initial cost.

$$\frac{R}{C_I} = \text{"Percent" overrun}$$

$$\frac{R_p}{C_I} = \text{"Percent" overrun prorated}$$

$$\frac{Q}{C_I} = \text{"Percent" Quantity changes.}$$

Finally,

$$\frac{R_p}{C_I} = \frac{R}{C_I} \cdot \frac{Q/C_I}{1 + Q/C_I}$$

Figure 2

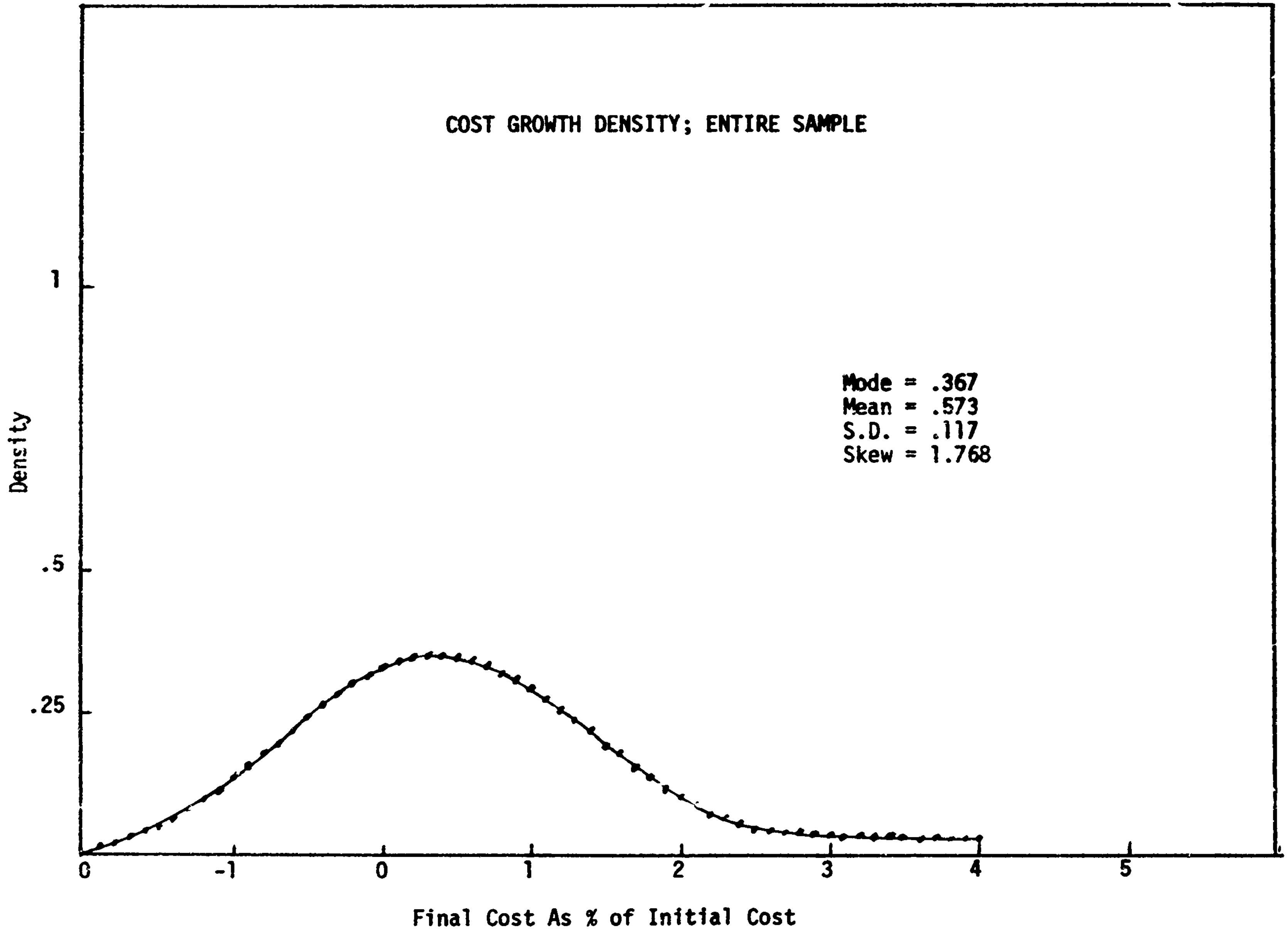
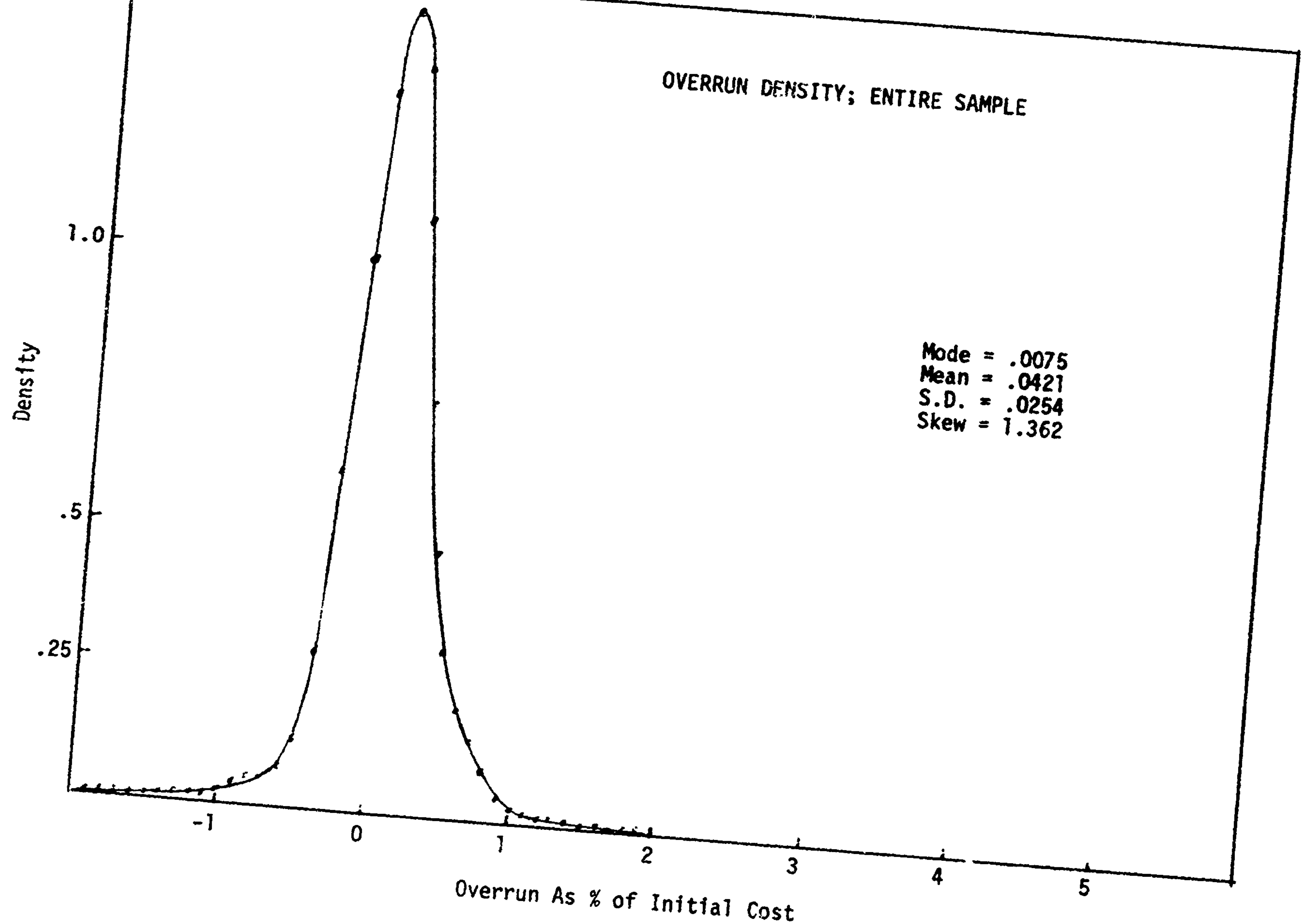

COST GROWTH DENSITY; ENTIRE SAMPLE
Mode = .367
Mean = .573
S.D. = .117
Skew = 1.768
Density
1
.5
.25
0
-1
0
1
2
3
4
5
Final Cost As % of Initial Cost

Figure 3



OVERRUN DENSITY; ENTIRE SAMPLE
Mode = .0075
Mean = .0421
S.D. = .0254
Skew = 1.362
Density
1.0
.5
.25
-1
0
1
2
3
4
5
Overrun As % of Initial Cost

Figure 4

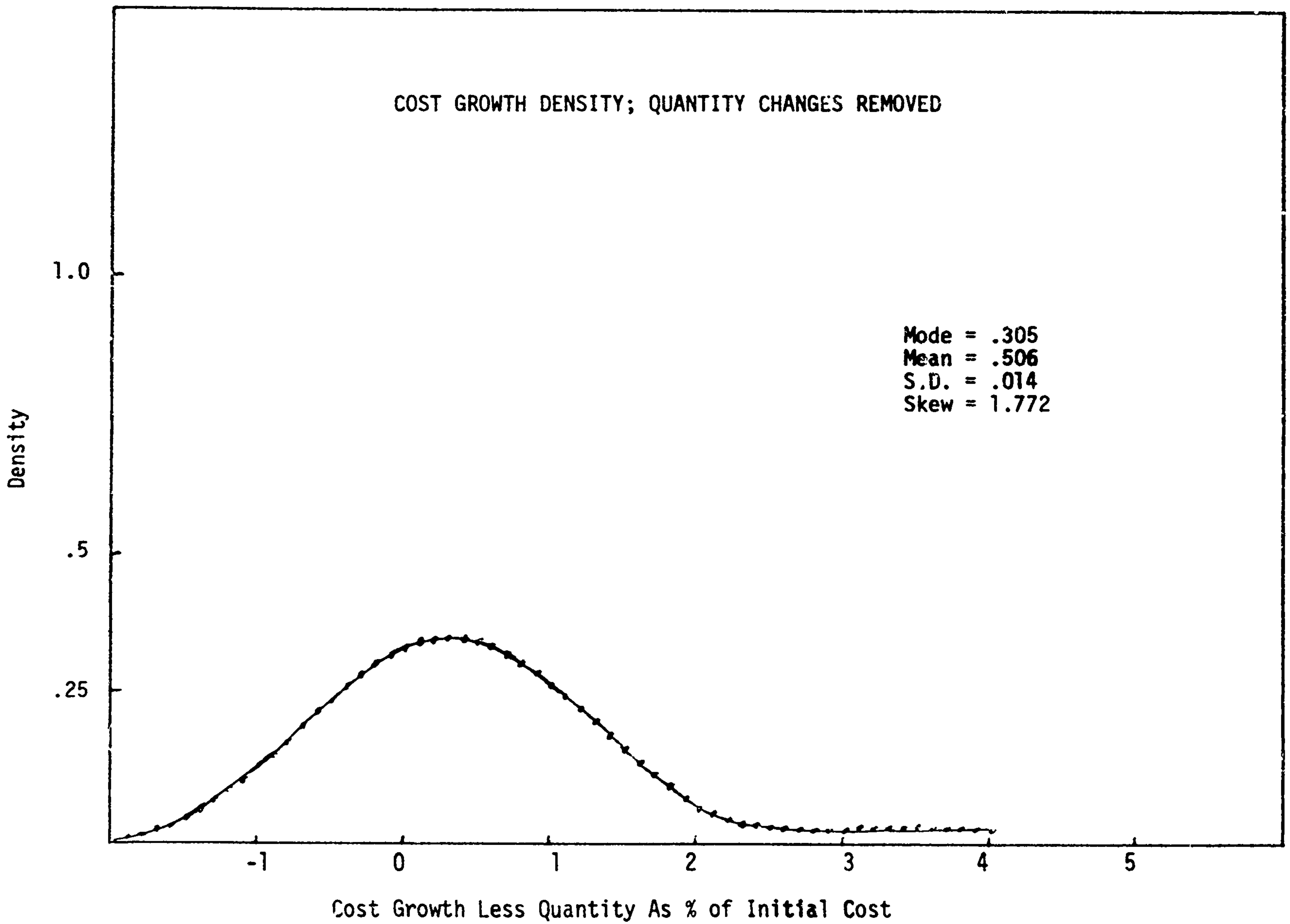
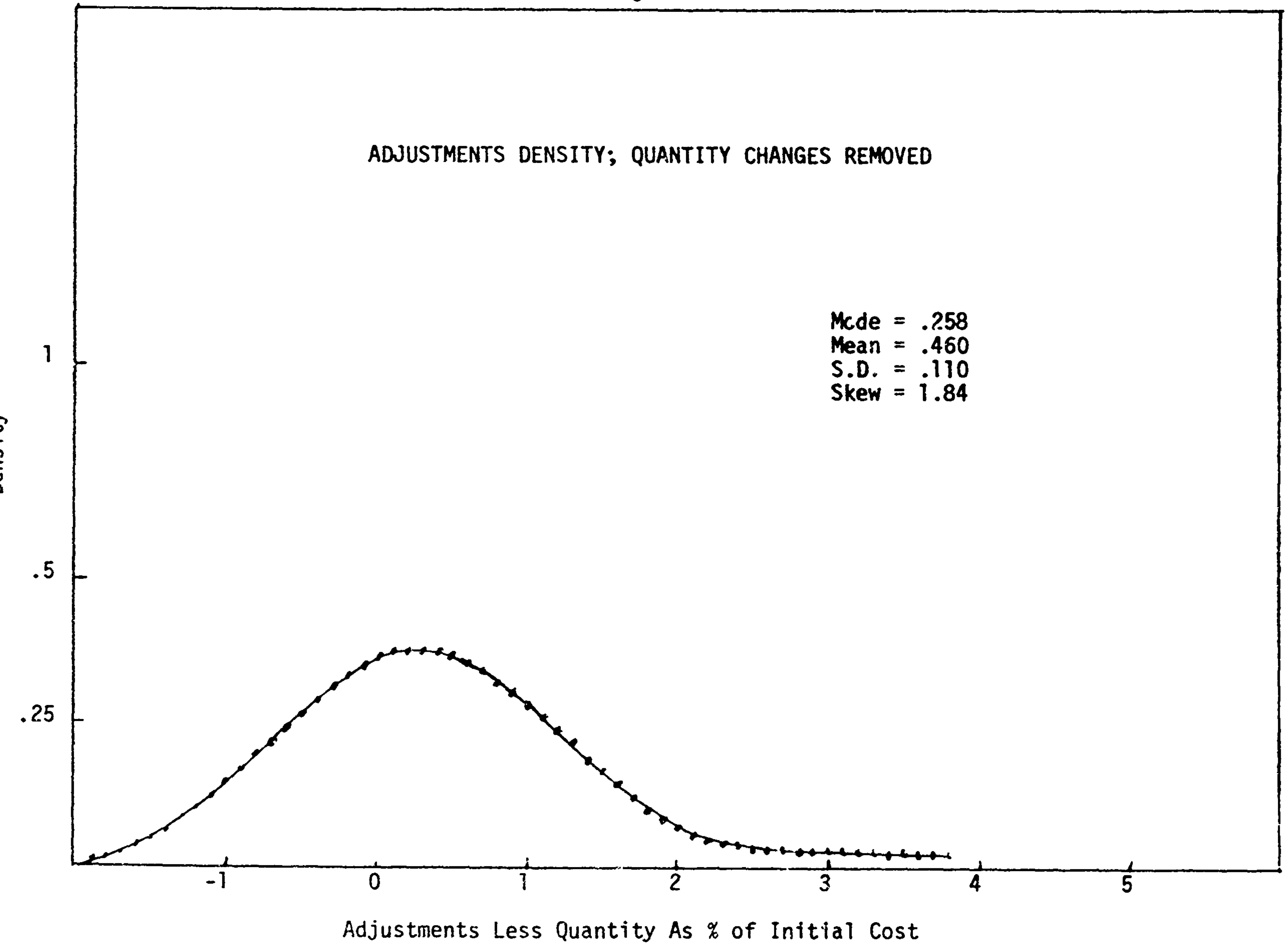

COST GROWTH DENSITY; QUANTITY CHANGES REMOVED
Mode = .305
Mean = .506
S.D. = .014
Skew = 1.772
Density
1.0
.5
.25
-1
0
1
2
3
4
5
Cost Growth Less Quantity As % of Initial Cost

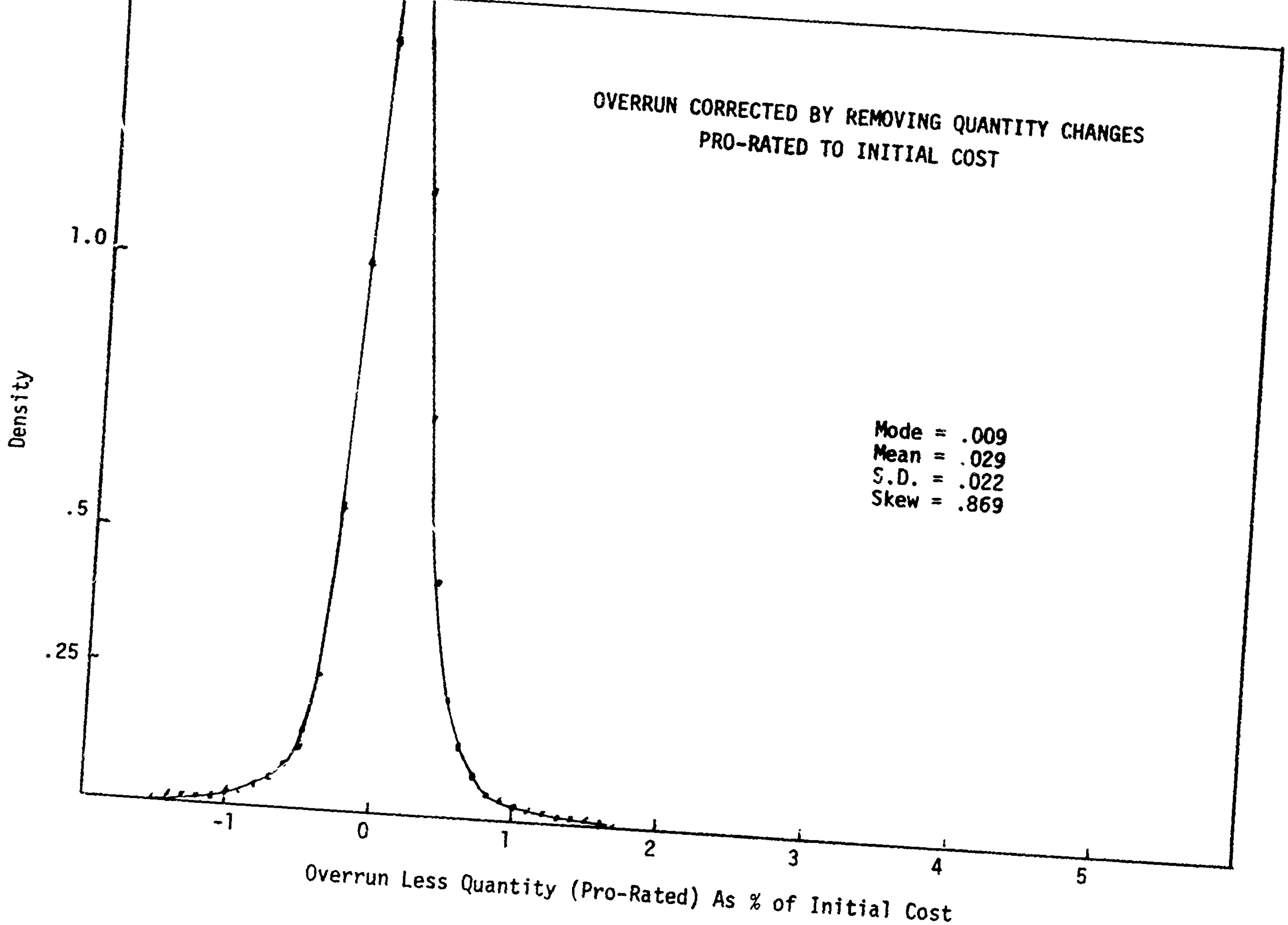
Figure 5


ADJUSTMENTS DENSITY; QUANTITY CHANGES REMOVED
Mcde = .258
Mean = .460
S.D. = .110
Skew = 1.84
Density
1
.5
.25
-1
0
1
2
3
4
5
Adjustments Less Quantity As % of Initial Cost

In a similar way, the overruns can be corrected or prorated on the basis of the adjusted cost, CA, or on the basis of the contract adjustments, $C_A - C_I$. The percent cost growth may then be corrected by subtracting the original overrun and adding the prorated overrun figure. Since only eight of the 53 CPIF contracts experienced quantity increases and of these, four experienced no contractual adjustments other than quantity changes while another had less than 1 percent quantity increases, only the overrun prorated to initial cost and adjustments were explored. In all cases, the overrun, contractual adjustments and cost growth exhibited a positive skew.

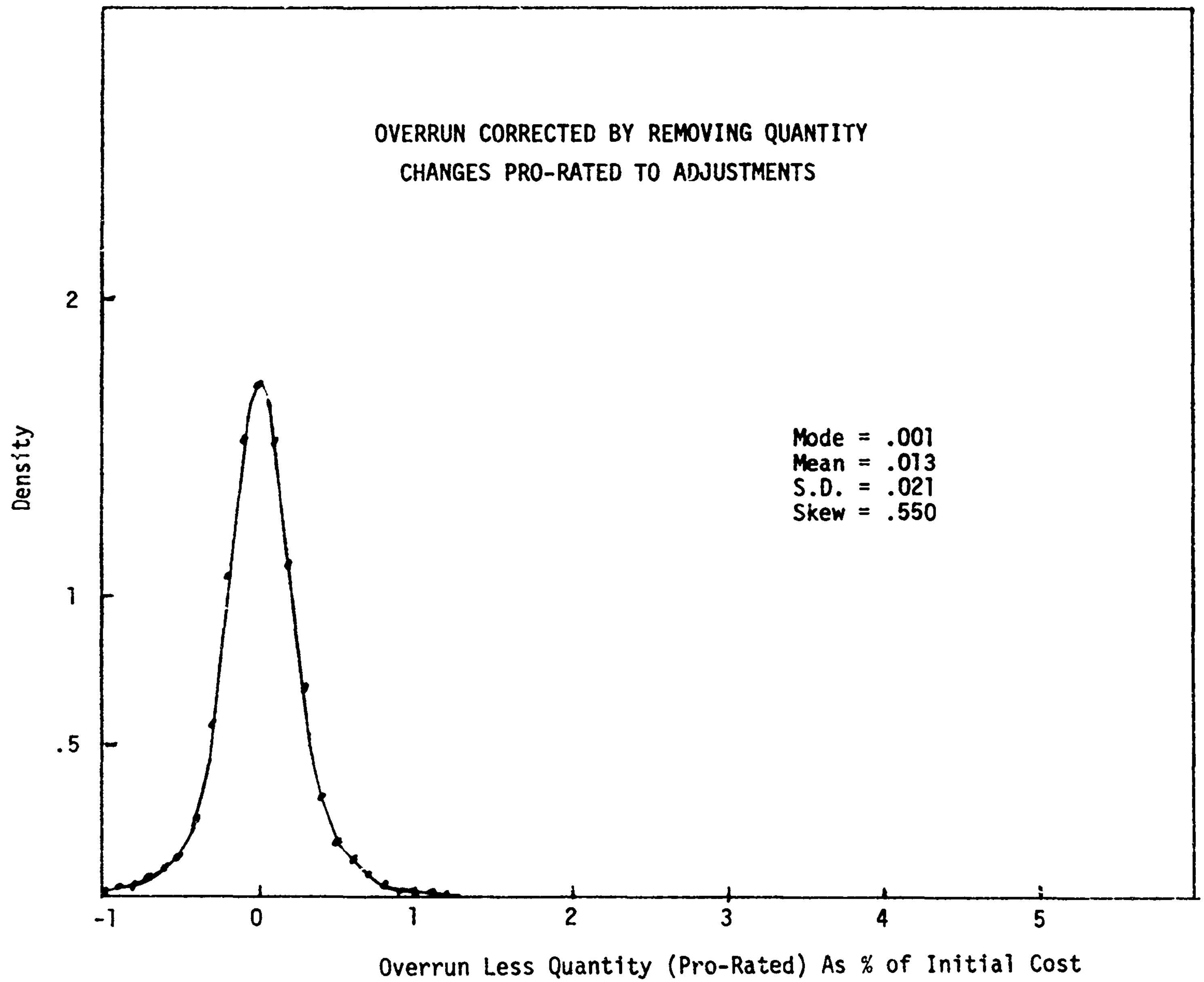
Figures 6, 7, and 8 give the overruns and cost growth corrected to initial cost and adjustments for the sample. Notice that the difference between mean and mode of cost growth is about 21 percent in both cases. Estimation of target costs with most probable costs should then result in approximately 21 percent cost growth on the average, even with no changes in scope or work definition. The remaining 26 percent or 27 percent cost growth is the average cost growth that would be experienced if expected costs and not most probable costs were used in procurements, and if the final cost is independent of the target cost.

Figure 6



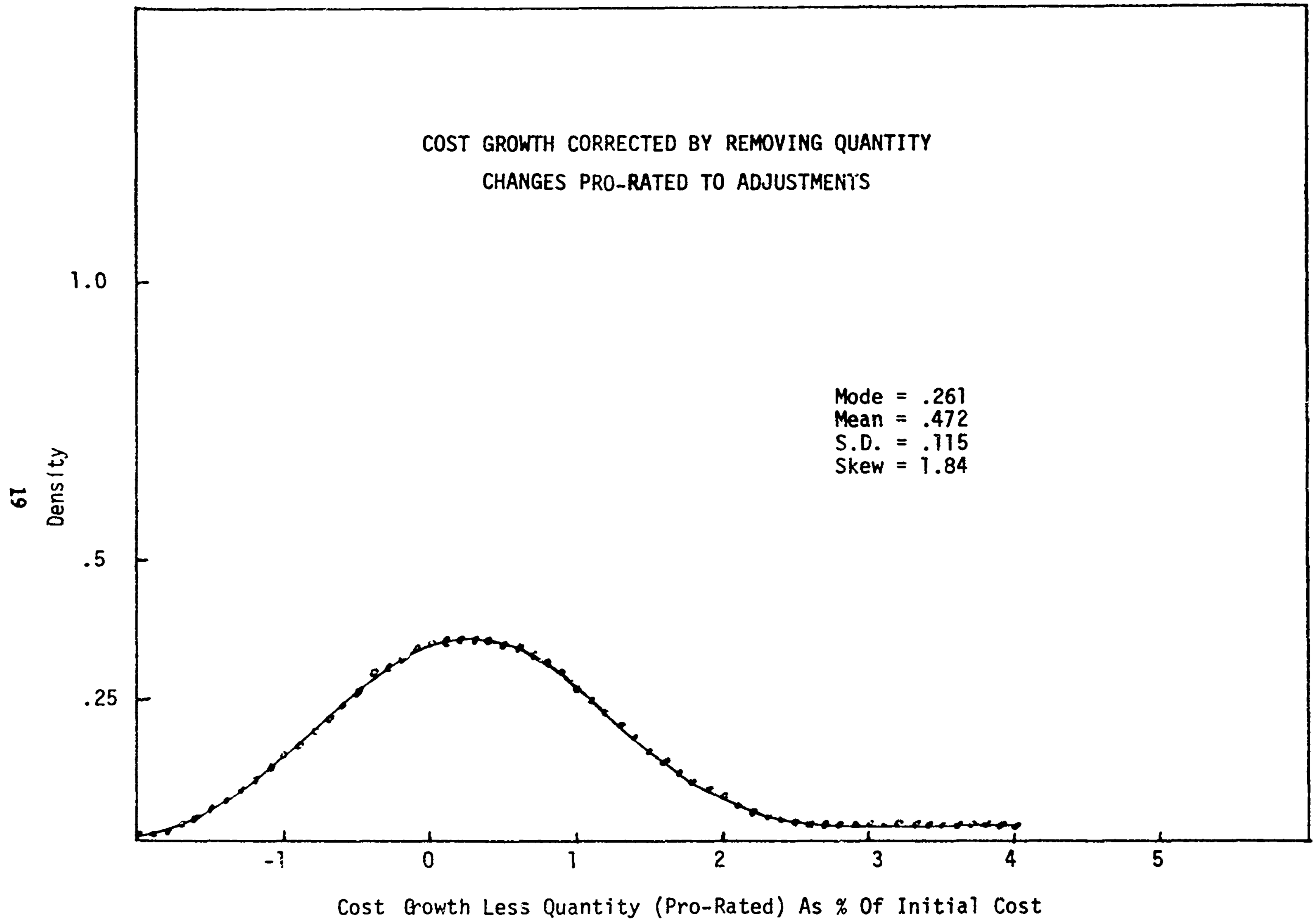
OVERRUN CORRECTED BY REMOVING QUANTITY CHANGES
PRO-RATED TO INITIAL COST
Mode = .009
Mean = .029
S.D. = .022
Skew = .869
Density
1.0
.5
.25
-1
0
1
2
3
4
5
Overrun Less Quantity (Pro-Rated) As % of Initial Cost

Figure 7



OVERRUN CORRECTED BY REMOVING QUANTITY
CHANGES PRO-RATED TO ADJUSTMENTS
Mode = .001
Mean = .013
S.D. = .021
Skew = .550
Density
2
1
.5
-1
0
1
2
3
4
5
Overrun Less Quantity (Pro-Rated) As % of Initial Cost

Figure 8



COST GROWTH CORRECTED BY REMOVING QUANTITY
CHANGES PRO-RATED TO ADJUSTMENTS
Mode = .261
Mean = .472
S.D. = .115
Skew = 1.84
1.0
.5
.25
Density
-1
0
1
2
3
4
5
Cost Growth Less Quantity (Pro-Rated) As % Of Initial Cost

C. The Cost Estimation Dilemma.

From a statistical point of view, the quality of an estimator may in some cases be improved by "underestimation," if quality means "closer more often." (Appendix II) This is frequently the case when the distribution has one mode and is skewed to the right as contract costs are. This measure of quality is not the same as the simple comparison of average with the actual value which is to be estimated (such as the measurement of cost growth), but is somewhat more complicated (Appendix II). Stated simply this means that using most probable costs for target costs is a statistically sound estimation procedure, even though it results in underestimating the costs. But average cost growth will be increased due to this underestimation. The following example is given to illustrate this point.

Suppose that cost analyst A, and cost analyst B use different techniques for estimating costs. Cost analyst A consistently underestimates costs by 4 percent to 6 percent, and on the average his estimates are 5 percent too low. On the other hand, cost analyst B is off by as much as 25 percent above or below cost, but his overall average cost estimate is precisely accurate. A contracting officer would almost certainly prefer the services of cost analyst A, while the chief budgeter for the Department of the Army might prefer to utilize cost analyst B in preparing the budget for the next fiscal year.

There is one other benefit to modal (under) estimation. There is good reason to believe that more realistic cost estimates obtained by increasing target costs will not reduce cost growth, [12, 13].

If final contract prices would remain the same regardless of the initial target cost, then overall cost growth could be eliminated. In fact, systematic overestimation would result in what would appear to be tremendous savings in tax dollars. Since this is not the way the procurement game is played [14], then maintaining the targets at lower levels will probably result in lower final costs but higher average cost growth.

The cost estimation dilemma is simply this: Underestimation of target costs provides (generally) more accurate estimates of costs and a lower base from which to operate, but higher average cost growth measurements. On the other hand, estimates which are accurate on the average have the disadvantage of the higher base but the advantage of lower average cost growth.

CHAPTER IV

THE EFFECT OF THE SHARE RATIO CONFIGURATION ON COST GROWTH

It was pointed out in Chapter I (Section B) that the S/R and RIE are determined by negotiation, so that it is reasonable to suppose that these two important elements of the CPIF type contract structure reflect the degree of uncertainty and risk inherent in the work statement of the contract. Unfortunately, the S/R and the RIE are not independent, and in fact the lower end point of the RIE depends quite strongly on the S/R since, by law, the profit may be no more than 10 percent of target cost (or 15 percent for R&D contracts; ASPR 3-405.6). The fact that the minimum target and maximum profit figures as a percent of target cost are almost always 6 percent, 8 percent and 10 percent or 7 percent, 8½ percent and 10 percent, respectively, lends reasonable doubt to the assumption that these figures are arrived at by a strict negotiation process. Therefore, given a percent minimum, target and maximum profit the RIE is determined by the S/R.

In other words, the S/R and the RIE are too closely related to be treated as independent variables in a statistical analysis, so that this chapter is devoted to an investigation into the relationship between the share ratio and the elements of cost growth. An indirect analysis of the RIE will be described in the next chapter.

Let	C_I	=	Contract initial cost
	C_A	=	Contract adjusted cost
	C_F	=	Contract final cost
	Y	=	Percent contractual adjustments = $(C_A - C_I)/C_I$
	X_1	=	Contractor's percent share of O/R
	X_2	=	Difference between contractor's share of O/R and contractor's share of U/R.

The meaning of the variable X_2 is explained with the following example. Suppose that the share ratio for a contract is 70/30 for overruns and 80/20 for underrun. Then $X_1 = 30$, and $X_2 = 30-20 = 10$. In order to avoid ambiguity, if a contract had broken share ratio either for overruns or for underruns, then that contract was not used in this analysis.

The analysis involves the linear regression model, $Y = M + AX_1 + BX_2 + e$, where the error term e is assumed to have zero expectation and also assumed to be independent (between contracts).

Table II gives the results of this regression analysis. The analysis indicates a strong dependence of contractual adjustments on the variable X_2, but none on X_1; i.e., the magnitude of the S/R for overruns does not affect the amount of contractual adjustments but the "degree" to which the S/R is broken has a pronounced effect. (The U/R S/R is obviously highly correlated with the O/R S/R. Thus, the result should hold for U/R as well. This was, in fact, verified.)

TABLE II

Multiple Regression Analysis of Percent Adjustments (Less Quantity Changes) w.r.t. Share Ratio Information

Sample size = 46

$$Y = 33.1 + \overset{***}{.50}\, X_1 - \overset{**}{4.7}\, X_2$$

Standard error = 65.72

S.D. of X_1 = 1.30 $\bar{X}_1$ = 16.66

S.D. of X_2 = 2.70 $\bar{X}_2$ = 1.135

$t_1 = .39$ $t_2 = 1.75^{**}$

$t_{43,.05} = 1.68$

NOTE: In this report, one, two, or three asterisks will be used to denote estimated parameters which have been found to be statistically significant at the .1, .05, or .01 levels, respectively.

The result in Table II is surprising and suggests immediately that the same analysis be conducted with O/R as the independent variable. It was decided to conduct a correlation analysis between O/R and adjustments as well. The O/R regression analysis is given in Table III and the correlation analysis in Table IV.

TABLE III

Multiple Regression Analysis of Percent O/R (Prorated) w.r.t. S/R Information

Sample size = 46

$$Y = 12.0 - \overset{*}{.51}X_1 - \overset{(*)}{.37}X_2$$

Standard error = 16.0

S.D. of $X_1 = .32$ $\bar{X}_1 = 16.794$

S.D. of $X_2 = .64$ $\bar{X}_2 = -1.135$

$t_1 = -1.62$ $t_2 = -.57$

$t_{43,.05} = 1.68$

TABLE IV

Correlation Analysis Between O/R and Contractual Adjustments in CPIF Contracts

$r = .12$ $n = 46$ $t = .77$

$t_{44,.1} = 1.3$

*See Appendix II, cost growth report II.

Before elaborating on these results, it must be pointed out that this sample includes CPIF contracts with multiple incentives, i.e., schedule and performance incentives (as well as cost incentives), which involve the trade-off of dollars on the one hand, and time and performance characteristics (such as air speed and weight) on the other hand. It is impossible to include these incentives in the present model because of the lack of data.

It was decided, therefore, to conduct a regression analysis on only those contracts in the sample with incentives relating to cost but not relating to schedule or performance, or on those contracts which did have multiple incentives and for which the exact amount of the profit, adjustments and overrun due to the cost portion of the incentive was given.

There are only 29 K's in the sample which satisfy one or the other of these restrictions, and of these 29, only six had a "broken" S/R. Since "high risk" contracts tend to have multiple incentives, it is not unlikely that this subsample is the "low risk" part of the original sample. Since contractor's behavior is highly dependent on the amount of his risk, and the contractual limitation on his behavior is dependent on the inherent risk involved, a risk factor or measurement must be introduced to the regression model in order to retain a reasonable degree of reliability on the inferences. No such risk measurement is available at this time. Hence, the value of this secondary analysis as a means of improving the information derived from the original regression analysis is open to question.

The result of the analysis involving only the "cost" contracts is given in Tables V and VI. None of the regression coefficients is statistically significant even at the .1 level. That is, with this data, no relationship between O/R or adjustments and S/R or RIE is detectable even at the 10 percent level of significance.

A schematic illustration of the first regression analysis is given in Figure 9. The sketch labeled (b) in figure 9 represents an unbroken share ratio. As the S/R becomes more "concave" in the upward direction, as depicted in (a), the tendency toward more contractual adjustments increases. As S/R assumes a profile which is "concave" in the downward direction, (c) there corresponds a decrease in the amount of contractual adjustments.

Sketch (e) is again supposed to represent the average S/R profile. As the slope in the share ratio decreases, (corresponding to a decrease in the contractor's share of the overrun and underrun) there is an increased tendency to overrun, while an increase in this slope is accompanied by a decrease in overruns, or an increase in underruns.

The profile in sketch (d) is characteristic of contracts of higher risk. Therefore, the indication is that the high risk contracts produce more overrun than lower risk contracts.

Sketches (a) and (c) characterize contracts with low target cost and high target cost, respectively, with respect to the expected final cost [6; 62–64, 81–87]. Sketch (a) is then representative of the "buy-in" situation. One would expect to experience more than the average number of contractual adjustments in this situation.

TABLE V

Regression Analysis of Percent Adjustments (Less Quantity Changes) for "Cost Only" Incentives

Sample size = 29

$Y = 23.2 + .554\ (X_1) - .08\ (X_2)$

S. Err. = 43.3

S.D. $X_1 = .967$

S.D. $X_2 = 2.19$

$t_1 = .572$ $t_2 = -.04$ $F_{3,26} = 5.6$***

$F_{3,26,.01} = 4.64$

TABLE VI

Regression Analysis of Percent O/R (Prorated) for "Cost Only" Incentives

Sample size = 29

$Y = -6.8 + .34\ X_1 - .017\ X_2$

Standard Error = 162.8

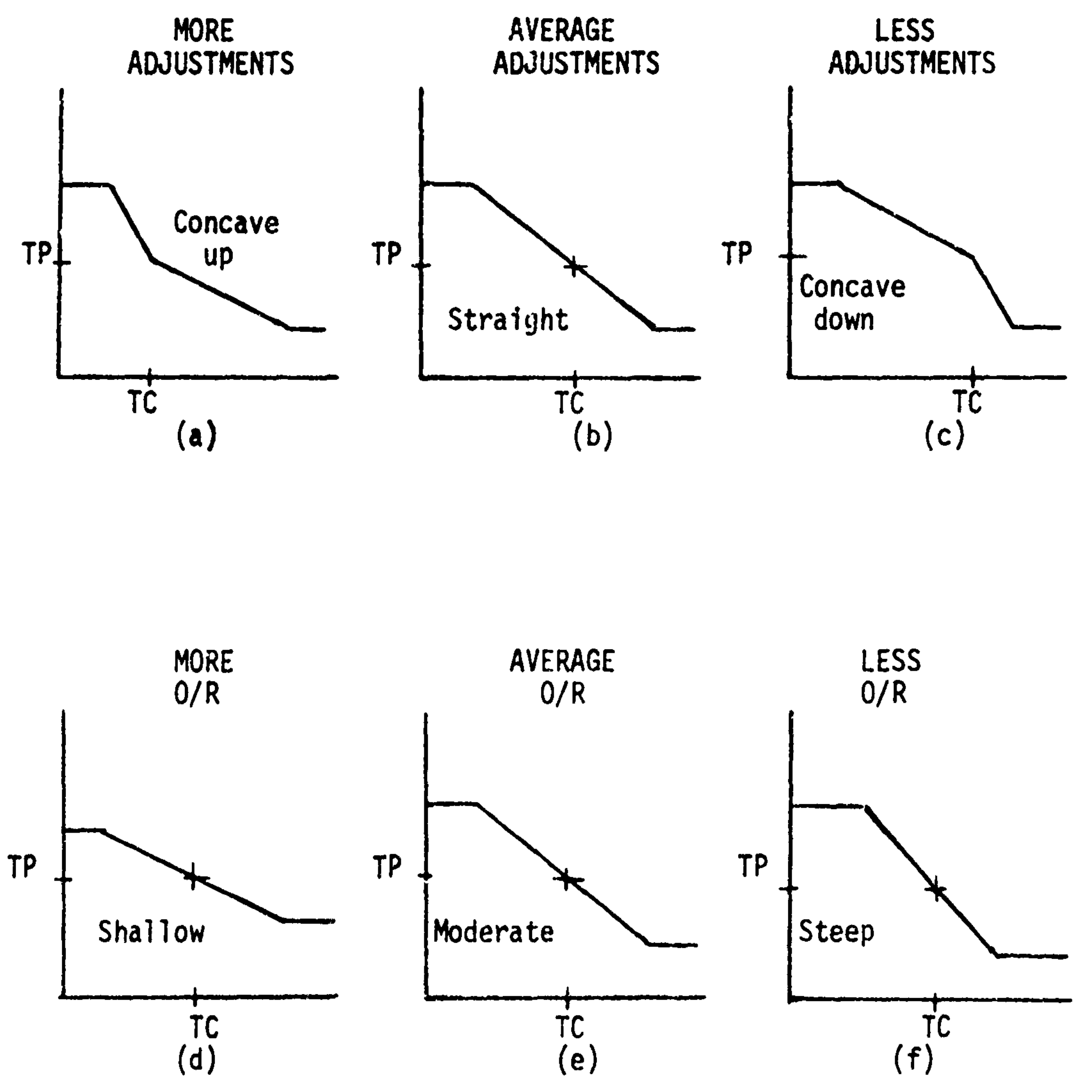
S.D. $X_1 = 3.64$

S.D. $X_2 = 8.12$

$t_1 = 1.19$ $t_2 = .027$ $F = .32$

$t_{26,.1} = 1.3$

SCHEMATIC ILLUSTRATION OF RELATIONSHIP BETWEEN DEGREE OF SHARE-RATIO BREAK AND BETWEEN SHARE RATIO AND OVERRUN



TC = Target Cost; TP = Target Profit

FIGURE 9

As an aid in drawing firm conclusions from this analysis, it would be helpful to know whether the broken S/R is caused by a decrease in contractor's S/R for O/R or an increase in his S/R for U/R or from both. In an effort to determine this, the average S/R for the contracts with unbroken S/R was compared, with those contracts that had a broken share ratio both for O/R and U/R, by means of a t-test. The results, given in Table VII indicate that only the S/R for U/R, in the "broken" case, is significantly different from the average "unbroken" S/R. In other words, the broken S/R is used more as a device for increasing the contractor's reward for underrun, rather than increasing his penalty for any overrun.

Note, that the standard deviation of the broken S/R for O/R is much lower than the standard deviations for the other two cases. Even though this difference (or ratio) of variances was not found to be significant at the 10 percent level with the usual F-test, it did seem advisable to calculate and display the Behrens-Fischer t-statistic along with the usual "pooled" t-statistic. Neither of these t-values approach significance at any "reasonable" level.

TABLE VII

Comparison of S/R's for the Broken and Unbroken Cases.

Type	Ave.	n	Var.	t (D.F.)
Unbroken S/R	17.2	37	64.9	–
Broken S/R (O/R)	15.8	9	18.0	1.1 (30) Behrens-Fischer .76 (44) Pooled
Broken S/R (U/R)	22.8	9	59.6	-1.87** (44)

$t_{44,.05} = 1.68$

The interpretation of the collective analysis presented in this chapter seems formidable. The main findings are as follows:

(1) There is a significant increase in contractual adjustments when the S/R for U/R is increased, and (2) a significant decrease in O/R (or increase in U/R) corresponding to an increase in the contractor's S/R. Finally, (3) it was found that contractual adjustments and O/R are statistically independent and (4) no significant effects were detected on adjustments and O/R due to the S/R when only the cost portion of the O/R and adjustments are considered.

The results of the analysis presented in this chapter are not unexpected, and the experienced procurement analyst would be the first to assert that. This chapter offers a statistical confirmation of the relationship between the incentive structure and risk in CPIF contracts.

CHAPTER V

THE RELATIONSHIP OF PROFIT TO SHARE RATIO AND RANGE OF INCENTIVE EFFECTIVENESS

In the introductory statement of Chapter IV, it was explained why a direct analysis of the RIE is considered inappropriate. The indirect analysis in this chapter will first of all attempt to determine how the final costs are distributed with respect to the RIE and especially, how this distribution appears with respect to those contracts with final cost outside the RIE.

There is one other indirect analysis suggested by this approach in conjunction with the result of Chapter II. Since the correlation between negotiated contractor's incentive share and that computed with final cost figures is quite low, is there any disparity between the contractor's actual profit and that which would be produced by the share line? The low correlation mentioned above does not necessarily imply that this should be the case but, in fact, if there are many final costs in the sample which are outside the RIE, this might be expected.

This chapter then begins with an examination of the contractor's expected profit as related to the RIE. In order to do this, it was decided to normalize the incentive S/R and RIE so that the TC, TP, Max P, Min P, and upper and lower ends of the RIE have the same value for each contract. This was done for all contracts which had no break in the S/R either above or below the TC and for which the complete S/R and RIE was available for the cost portion of the contract only. This "normalized" S/R line is shown in figure 10. The purpose of this data transformation is so that a sample of more than one observation could be subject to analysis.

NORMALIZED SHARE LINE USED IN THE

PIECE-WISE LINEAR REGRESSION; PROFIT VS. COST

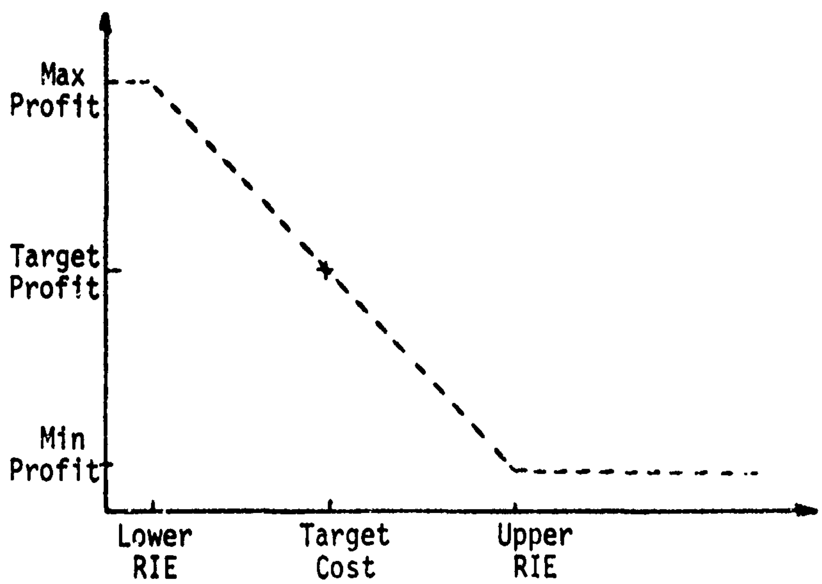


FIGURE 10

Figure 11 shows the transformed data for the 30 observations. The degree of scatter about the normalized share line appears quite high. Several reasons for this are alluded to in Chapter II in connection with the correlation analysis of negotiated S/R with respect to the estimated S/R. The one which would appear to have the most marked effect is the existence of multiple incentives; i.e., incentives which reward (or penalize) the contractor for completing the contract ahead of (or behind) schedule or which reward (or penalize) the contractor for delivering a product which performs better (or worse) than required by the contract.

Therefore, the contract cost figures were "cleaned up" so that they would reflect only payments, penalties, profits, and costs made under the cost portion of the incentive. Figure 12 shows this "cost-portion-only" data. There appears to be no significant change in the degree of scatter, but one of the data points has moved from outside the RIE (on the high side) to within the RIE. Thus only four of the 30 points are above the RIE; and two lie below the RIE.

It would be desirable at this point to test the hypothesis that this data could have been generated by the normalized cost share line, and also to test similar hypotheses for the two cases, within and outside the RIE, separately. Since there are only two points below and four points above the RIE, and these points are generated by a "different" straight line model than are those within the RIE, a statistical method which analyzes several linear models in combined form as a "piecewise-linear" model is required.

A search for such a model was conducted in all of the available literature, but none was found. Therefore, this statistical technique was developed within APRO. (See Appendix III for details or reference [9] for a complete description.)

The Piecewise Linear Model (PLM) employed in this analysis is as follows,

$$y = \begin{cases} a_1 + b_1 X & 1 \leq x \leq 2 \\ a_2 + b_2 X, & X \geq 2 \end{cases} \quad \text{, subject to the constraint,}$$

$a_1 + 2b_1 = a_2 + 2b_2$, where y represents final normalized profit and X represents final normalized cost.

It should be pointed out that the two data points to the left of the y-axis result from underruns which brought the final cost below the lower limit of the incentive range, and hence the contractor received the maximum profit. Whether all points in this region would be exactly on the theoretical profit line or not, cannot be ascertained. Therefore, only the 28 points which correspond to final costs within the RIE or above the upper limit of the RIE were subjected to analysis.

The normalized share line is depicted in figure 12 by the solid line, and the estimated "expected share line" is represented by the dashed line. The regression analysis (table VIII) indicates a significant difference between the parameter of the solid and the dashed line at the 5 percent level.

It is important to know which of the parameters contributes to the significance indicated by the large F-value. It is equally important in this particular analysis, to know which, if any, of the parameters is not significant. To accomplish this, the parameters were tested individually. The results are given in Table IX.

For future reference, it is mentioned that the average negotiated S/R for the contracts within and above the RIE are 17.5 percent and 10 percent, respectively. The percent increase in target cost due to contract modifications, less O/R and quantity changes are 25.5 percent and 2.3 percent for contracts with final costs within and above the RIE, respectively. Finally, the percent O/R was 1.4 percent and 45 percent respectively, for those contracts with final costs within and above the RIE.

THE DATA WITH NO QUANTITY CORRECTION

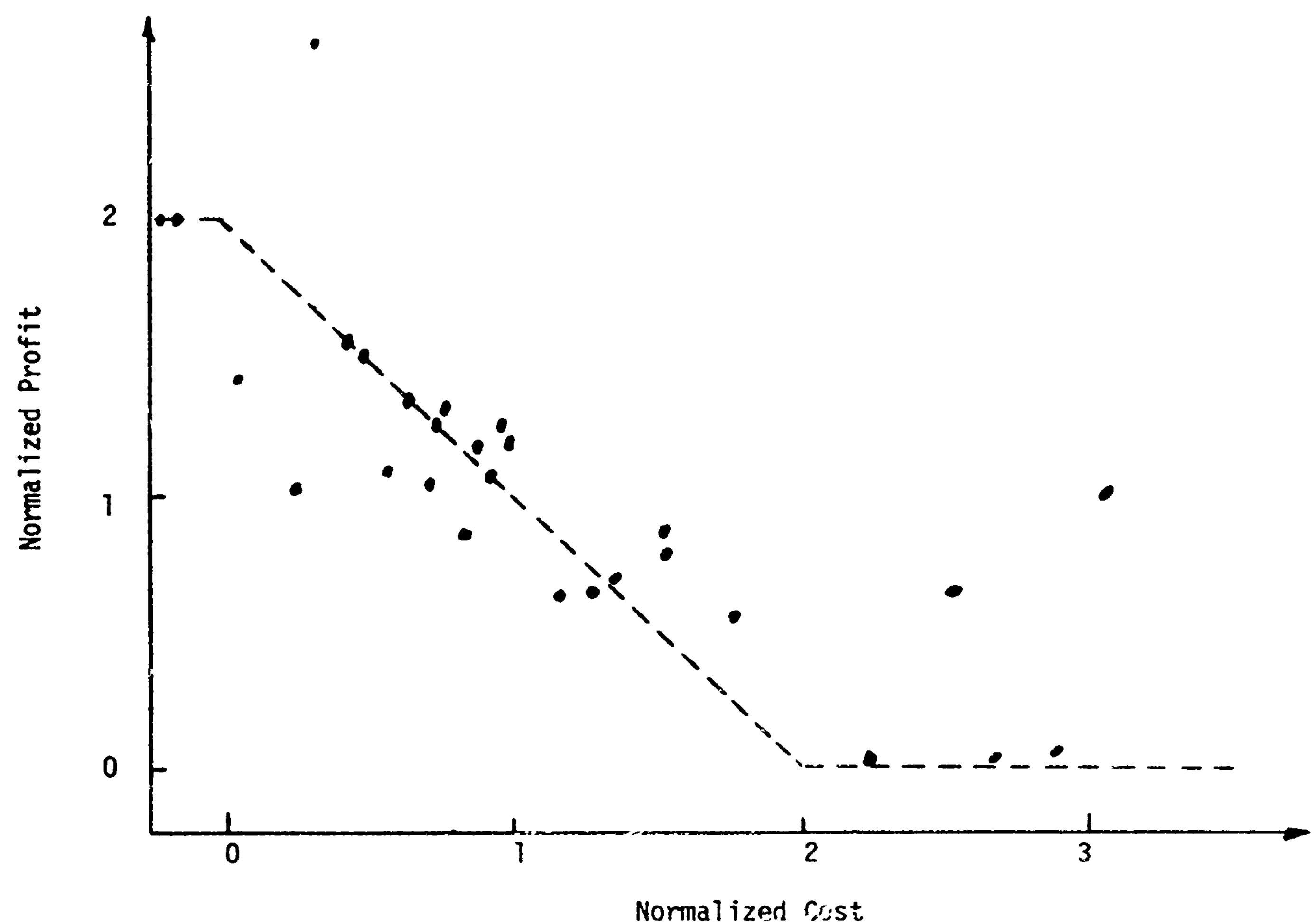


FIGURE 11

THE DATA WITH QUANTITY CHANGES CORRECTED TO INITIAL COST

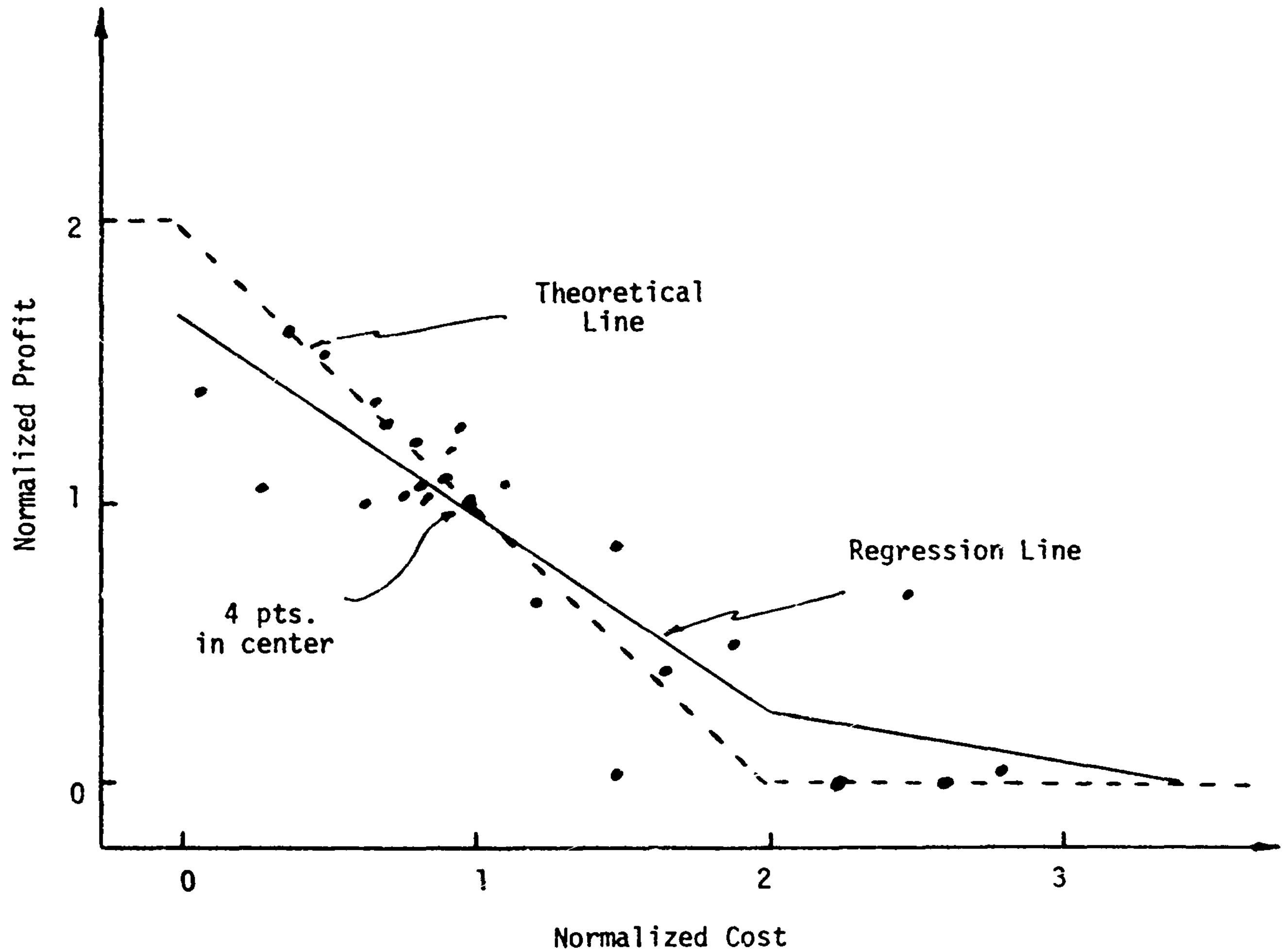


FIGURE 12

TABLE VIII

Piecewise Linear Regression Analysis of Normalized Cost and Profit.

Test on Entire Model.

$$y = \begin{cases} 1.0185 + .7004\,(x - .9292) & , x \leq 2 \\ .1819 - .157\,(x - 2.55) & , x \geq 2 \end{cases}$$

$n = 28 \qquad m_1 = 24 \qquad m_2 = 4$

Standard Error = .237

Ho: $b_1 = -1,\ a_2 = o,\ b_2 = o.$

$F = 3.16^{**}$

$F_{3,25,.05} = 2.99$

The F-ratio is significant at the 5 percent level. In an effort to determine which of the parameters contribute the most to this significance, the parameters were tested individually. The results are given in Table IX.

TABLE IX

Piecewise Linear Regression and Hypothesis Test on Individual Pieces of the Model, With and Without Constraints.

$n = 28$, $m_1 = 24$, $m_2 = 4$.	
Ho: $b_1 = -1$.	(with constraint)
$F = 6.9^{**}$	$F_{.05,1,25} = 4.24$
Ho: $a_1 = 1$	(no constraint, a_1 corrected for mean)
$F = 3.646^{**}$	$F_{.05,2,22} = 3.44$
Ho: $a_2 = o$, $b_2 = o$	(with constraint)
$F = 3.15$	$F_{.05,2,25} = 3.39$
Ho: $_2 = o$	(with constraint)
$F = .139$	
Ho: $b_2 = o$	(with constraint)
$F = .302$	

It is clear that the slope estimate obtained from the points corresponding to final costs which are within the RIE is causing the significance. The indication is that the share of both O/R and U/R which the contractor is actually receiving is less than the negotiated share ratio. This does NOT mean that the Government changes or disregards the terms of the contract. It DOES mean that perturbations are brought about which are not (completely) governed by the contractual agreement but which do change either the cost or profit or both. One example of this type of change is the contract renegotiation. (See also Chapter I, Section C.)

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The analysis described in Chapters II through V was performed to answer the questions posed in Chapter I, Section C. Although several of the questions are answered very directly, several of them will require additional explanation. The major findings will first be summarized so that they may be easily referred to during a reading of the immediately following section.

A. Major Findings.

1. The estimated correlation coefficient between negotiated share ratio and the share ratio estimated from the final cost information from CPIF contracts is .15 with a t value of .79. Even when the data is restricted to those contracts which experienced an overrun and which had an "unbroken" negotiated share ratio, the estimated correlation coefficient is less than .25 with a t-value of about .75.
2. When the cost growth average and mode are both computed as a percent of initial target cost, the average was found to be 20 percent greater than the mode. Since estimates of "most probable cost" (i.e., the mode) are used as contract target costs, and since we measure "average" (i.e., mean) cost growth, there appears to be a built-in cost growth base of 20 percent on all CPIF contracts.
3. The data yielded a significant positive correlation between the contractor's share of underrun and contractual adjustments.
4. The data indicated a statistically significant, negative correlation between O/R and the contractor's share for overruns.
5. Contractual adjustments and overruns are statistically independent.
6. The share of both the underrun and the overrun which the contractor actually receives is, on the average, less than the negotiated share, within the range of incentive effectiveness.

B. Discussion.

Some of the questions posed in Chapter I, Section C, involve the intent or desires of contractors. Since intents and desires are not measurable, the questions which involve them are not amenable to direct statistical analysis. For this reason, and because questions of this type are somewhat accusatory toward the contractor, it would seem that the only way to answer the questions with a high degree of confidence would be in a court of law. This is clearly impossible within the confines of this project, so a search for indicators of an indirect sort has been employed in this study.

It is natural to assume that a contractor ceases to attempt to control costs when he loses the incentive to do so; i.e., when the contract cost has moved to a point outside the range of incentive effectiveness (RIE). There is an indication of this in the analysis.

The data in figure 12 indicate that the spread of final costs which were above the RIE is large. Furthermore, the negotiated S/R for these contracts was only 10 percent as opposed to the 17.5 percent for the others.

A positive identification of the buy-in also proved elusive. The phrase "buy-in" refers to the situation that occurs when a contractor bids so low on a particular proposal that he is virtually insured of being awarded the contract, thereby literally "buying" the contract (away from competition).

One indication of the buy-in is an S/R configuration similar to that shown in figure 9 (a); that is, an S/R which is concave upwards and a target cost which is nearer to the lower end of the RIE than to the upper end. The regression analysis in Table V shows that as the S/R becomes more concave upward, the number of contract adjustments increases. Table VII indicates that the concavity is most affected by the U/R portion of the S/R, and since underrun increases with the S/R, the existence of the "buy-in" pattern seems to be verified.

The analysis of Chapter III indicates that the use of estimates of the most probable contract cost for use as the target cost, as opposed to using estimates of the expected cost, may produce as much as 21 percent cost growth.

The regression analysis reported in Chapter IV did not provide evidence of a relationship between contractual adjustments and the contractors negotiated share of the overrun, although, the O/R was found to decrease with an increase in the contractor's negotiated share. It is felt that this is produced by the "buy-in" phenomena described above rather than uncertainty.

A relationship was found to exist between the degree of break in the share ratio and the amount spent on contract modifications. When the break in the share line produced a "concave upward" configuration (see figure 9, Chapter IV) the amount spent on modifications was highest. But as the share line assumed the form of a straight line and passed on to the "concave downward" configuration the modifications steadily decreased on the average. If the overrun followed an identical pattern, then this effect could reasonably be attributed to "uncertainty," while if O/R were low (i.e., high U/R) whenever the share line configuration was concave upward, then the aforementioned effect could be assumed to be due to the "buy-in" or "low initial cost-high modifications" type of contractor behavior. No dependence of O/R on either the share line configuration or on contract modifications was detected. Since both of these should produce the effect on contract modifications noted above, while their effects on O/R are counteractive, the overall result is a general confirmation of the possible existence of both causes—uncertainty and buy-in.

The "piecewise" linear regression which is described in Chapter V, was conducted in an effort to determine whether or not average profits for contracts with final costs above the RIE are high enough to justify a recommendation to extend the upper end of the RIE. The analysis, however, indicated that the average profit in this final cost range did not differ

significantly from that which was negotiated. It was also discovered that the average profit for contracts experiencing O/R and U/R were, respectively, greater than and less than the profit dictated by the negotiated share line within the RIE. It is pointed out that this effect could not be entirely due to noise in the data but must be at least partially generated by a systematic mechanism.

In any case, there is no apparent reason for recommending the extension of the RIE.

Chapter II provides a positive basis for rejecting the use of estimated share ratio obtained from final contract cost figures instead of the negotiated share ratio, when the latter is not available to the analyst.

C. Recommendations.

The results of this study are of such a nature, that clear-cut policy recommendations would be exceedingly difficult to make. Therefore, the recommendations are pointed at either senior Army policy makers for consideration in policy formulation, or else for analysts and researchers who offer advice to the policy makers or who conduct investigations in logistics problems.

1. It is recommended that senior DA procurement analysts be made aware of the relationship between the mean and the mode in CPIF cost data. The obvious but simplistic recommendation that "expected" costs should be used instead of "most probable" costs will be avoided here, because of the possible effect of the resulting higher target costs on the final costs. Any further policy recommendation will require more study tempered with sound procurement judgment.

2. It is recommended that procurement analysts be made aware of the disparity between the contractors negotiated share of overrun and underrun and the (smaller) share which he receives on the average. A more extensive recommendation will not be given since this disparity works to the advantage of the Government in the underrun situation. There is also the possibility of the existence of hidden "trade-offs" between the Government and the contractor, which are not measurable from the data, but which work to the benefit of the Government.

3. It is recommended that studies performed within the Department of the Army involving the affect of the share ratio in CPIF contracts on cost growth should use the negotiated share ratio obtained from the contract files. Analysis which is based on share ratios estimated from the final cost data may be highly inaccurate.

Appendix I

This appendix is a description of the density estimator used in this report which was taken directly from the paper entitled "On Estimation of a Probability Density Function and Mode" by Emanuel Parzen, which appeared in the Annals of Mathematical Statistics, Volume 33, 1962, [11]. Only the barest details are given here for brevity. The interested reader is referred to the original paper.

Suppose that X is a R.V. with density function f(x), and $x_1, x_2, \ldots, x_n$ is a random sample of observations of X. Parzen proposes a general class of weighting functions K(y) to estimate f(x) as follows:

$$f_n(y) = \frac{1}{nh}\sum_{i=1}^{n} \frac{K(y-x_i)}{h},$$

where $f_n(y)$ is the estimate of f(y), and h = h(n) is a function of the sample size; Parzen shows that $f_n(y)$ is asymptotically unbiased if K(y) is bounded, if the total integral of K(y) converges absolutely, and if $\lim_{n \to \infty} h(n) = 0$,

$$\lim_{y \to \infty} |yK(y)| = 0, \qquad \int_{-\infty}^{\infty} K(y)\,dy = 1.$$

Further, the variance may be estimated from

$$\lim_{n \to \infty} \left(nh \operatorname{Var}\left[f_n(x)\right]\right) = f(x)\int_{-\infty}^{\infty} K^2(y)\,dy$$

if f(x) is continuous at x.

Parzen suggests seven specific forms for K(y). A number of Monte Carlo simulation runs indicated that n = 12, 24, and 36, the function given by

$$K(y) = \frac{1}{2\pi}\cdot\left(\frac{\sin(y/2)}{y/2}\right)^2$$

is noticeably superior to the others. This one, incidentally, also has the smallest variance as reported in Parzen's paper, and is approximately

$$\operatorname{Var} f_n(x) \simeq \frac{1}{3\pi nh}\cdot f(x)$$

The sample mode, θ_n, is obtained by finding $f_n(\theta_n) = \max(-\infty < x < \infty)\, f_n(x)$.

In this study, a simple grid procedure was used and θ_n was obtained to within .0001 with three iterations of 21 estimates of $f_n(x)$.

The sample mode (as are the values of $f_n(x)$) are asymptotically unbiased and asymptotically normally distributed. The variance is approximately:

$$Var(\theta_n) \approx \frac{.01}{n h^3}\left(f(\theta_n)/f''(\theta_n)\right)^2$$

Since the estimate involves the second derivative of f(x), and the distributions involved in this study were rather sharply peaked, estimating the second derivative is difficult. Attempts were made at modifying Parzen's estimator by estimating

$$(f(x+h) - f(x))/h$$

for the first derivative and similarly for the second derivative. These estimates were rather unstable, but not completely useless.

The h value used in this study is taken from studies performed in the Statistics Department at VPI & SU at Blacksburg, Virginia (cf. [1]).

$$f(y) = \frac{1}{2\pi n h}\sum_{i=1}^{n}\left(\frac{\sin G_i}{G_i}\right)^2 ,$$

$$G_i = (y - x_i)/2h ,$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i ,$$

$$h = n^{-1/5}\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]^{1/2} .$$

Appendix II

The purpose of this appendix is to indicate how an estimator which underestimates, on the average, can be better than another estimator which does not underestimate or overestimate on the average. A specific example will be given.

Let $X_1, X_2, \ldots X_n$ represent a random sample of the r.v. X with unknown parameter θ, and frequency function $f(x; \theta)$. Further, let $\hat{\theta} = g(x_1, \ldots, x_n)$ represent an estimator of θ. $\hat{\theta}$ is said to be an <u>unbiased</u> estimate of θ if its expected value is θ; i.e., $E(\hat{\theta}) = \theta$. The quantity $E\,(\hat{\theta}-\theta)^2$ is called the "mean squared error," MSE, and if $\hat{\theta}$ is unbiased for θ, this is also the variance of $\hat{\theta}$, or Var $(\hat{\theta})$.

The problem of comparing the quality of different estimators is bothersome, because there is no objective universally accepted measure of quality. When comparing two unbiased estimators, the better one is conceded to be the one with the smaller variance, because the average squared deviation of this estimator is smaller. In other words, it will be "closer more often." The logical extension of this idea is to simply compare the average squared deviation (MSE) disregarding the bias altogether. As a matter of fact, it can happen that the MSE of an unbiased estimator can be reduced by introducing a bias. This can be seen in the following development.

The MSE of an estimator, $\hat{\theta}$, can be written,

$$\text{MSE}\,(\hat{\theta}) = E\,[\hat{\theta} - E(\hat{\theta})]^2 + [E(\hat{\theta}) - \theta]^2 \qquad (1)$$

Now multiply the estimator $\hat{\theta}$ by the (as yet unknown) constant, A, to form the new estimator, $\tilde{\theta} = A\hat{\theta}$. Let $m = \text{MSE}\,(\tilde{\theta})$. One method of finding that value of A which will minimize m (if there is such a value), is to "set" the first derivative of m with respect to A equal to zero, and solve for A. Thus,

$$m = A^2\,\text{Var}\,(\hat{\theta}) + [A\,E\,(\hat{\theta}) - \theta]^2$$

$$\frac{\partial m}{\partial A} = 2A\,\text{Var}\,(\hat{\theta}) + 2E(\hat{\theta})\,[A\,E\,(\hat{\theta}) - \theta]$$

$$\frac{\partial^2 m}{\partial A^2} = 2\,\text{Var}\,(\hat{\theta}) + 2[E\,(\hat{\theta})]^2 > 0$$

$$A = \theta E\,(\hat{\theta}) / \left(\text{Var}(\hat{\theta}) + E^2\,(\hat{\theta})\right)$$

$$A = \theta E(\hat{\theta}) / E\,(\hat{\theta}^2) \qquad (2)$$

One example of an estimator which satisfies this last equation is the variance estimate for a normal r.v.

Let X be normally distributed with mean μ and variance σ^2 (both unknown). An unibased estimate of σ^2 is

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Since (n-1) S^2/σ^2 has the chi-square distribution with (n-1) degrees of freedom, then the variance of S^2 is $2\sigma^4/(n-1)$ and thus the second moment about the origin (from (1)), is $\sigma^4(n+1)/(n-1)$. Substituting these values into (2) yields:

$$\frac{\sigma^4}{\sigma^4(n+1)/(n-1)} = \frac{n-1}{n+1}$$

Therefore, the MSE of S^2 as an estimator of σ^2 may be reduced by multiplying it by the number (n-1)/n+1). In other words,

$$S'^2 = \frac{1}{n+1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

has smaller MSE than S^2, and S'^2 underestimates σ^2, because S^2 is unbiased and, therefore,

$$E(S'^2) = \frac{n-1}{n+1}\sigma^2 < \sigma^2$$

Finally, MSE $(S'^2) = 2\sigma^4/(n+1)$ which is less than MSE $(S^2) = 2\sigma^4/(n-1)$.

Appendix III

This appendix is a description of the piecewise linear model used in the analysis of Chapter IV. Full details may be obtained in Reference 9. The model is of the form

$$
y = \begin{cases} a_1 + b_1 (x - \bar{x}_1), & x \leq x^* \\ a_2 + b_2 (x - \bar{x}_2), & x > x^* \end{cases}
$$

subject to the constraint

$$
a_1 + b_1 (x^* - \bar{x}_1) = a_2 + b_2 (x^* - \bar{x}_2).
$$

The number x^* is assumed known.

It is convenient to employ matric notation. If the x-matrix is written:

$$
X = \begin{bmatrix} 1 & x_1 - \bar{x}_1 & 0 & 0 \\ 1 & x_2 - \bar{x}_1 & 0 & 0 \\ \vdots & & \vdots & \vdots \\ 1 & x_m - \bar{x}_1 & 0 & 0 \\ 0 & 0 & 1 & x_{m+1} - \bar{x}_2 \\ 0 & 0 & 1 & x_{m+2} - \bar{x}_2 \\ \vdots & & \vdots & \vdots \\ 0 & & 1 & x_n - \bar{x}_2 \end{bmatrix}
$$

and $\beta' = [a_1, b_1, a_2, b_2]$, then the model may be written:

$y = x\beta$, subject to $T\beta = o$, where $T = [1, x^* - x_i, -1, -x^* + x_2]$ and

$$
\bar{x}_1 = \frac{1}{m} \sum_{i=1}^{m} x_i \qquad \bar{x}_2 = \frac{1}{n-m} \sum_{i=m+1}^{n} x_i
$$

Notice that for $i \leq m$, $x_i < x^*$ and for $i > m$, $x_i > x^*$.

The method of LaGrangian multipliers applies to this minimization problem subject to the constraint. It is necessary then, to find the extreme value of

$$(y - X\beta)'(y - X\beta) + \lambda T\beta ,$$

where λ is the LaGrangian Multiplier. Differentiation with respect to β and λ yields the "constrained" normal equations:

$$X'X\tilde{\beta} + \tfrac{1}{2} T'\lambda = X'y$$

$$T\beta = 0$$

Solving for $\tilde{\beta}$ one obtains

$$\tilde{\beta} = (X'X)^{-1}X'y - \lambda/2\,(X'X)^{-1}T' ,$$

$$\lambda/2 = [T(X'X)^{-1}T']^{-1}[T(X'X)^{-1}X'y] .$$

If $\hat{\beta} = (X'X)^{-1}X'y$, the usual least squares estimate of β, then

$$\tilde{\beta} = \hat{\beta} - (X'X)^{-1}T'[T(X'X)^{-1}T']^{-1}T\hat{\beta} .$$

Obtaining the distributional properties of $\tilde{\beta}$ is straightforward but cumbersome (see reference.). It may be shown that the error sum of squares subject to the constraint is:

$$\widetilde{SS}_E = (y - X\tilde{\beta})'(y - X\tilde{\beta})$$

and if

$$SS_E = (y - X\hat{\beta})'(y - X\hat{\beta}), \qquad \widetilde{SS}_E - SS_E = T\hat{\beta}'[T(X'X)^{-1}T']^{-1}T\hat{\beta} .$$

Finally,

$$\frac{\widetilde{SS}_E}{\sigma^2} \sim \chi^2_{n-p+1}$$

For testing the hypothesis Ho: $T_0 \underline{\beta} = \underline{d}_0$ Subject to $T\underline{\beta} = \underline{0}$, Let

$$T^* = \begin{bmatrix} T \\ \hline T_0 \end{bmatrix}, \qquad \underline{d}^* = \begin{bmatrix} \underline{0} \\ \hline \underline{d}_0 \end{bmatrix}.$$

$$G = T\hat{\underline{\beta}}' \left[AA'\right]^{-1} T\hat{\underline{\beta}} \qquad G^* = \left[T^*\hat{\underline{\beta}} - \underline{d}^*\right]' \left[A^*A^{*\prime}\right]^{-1} \left[T^*\hat{\underline{\beta}} - \underline{d}^*\right]$$

Then the test statistic is

$$F = \frac{G^* - G}{SS_E} \cdot \frac{n - p + 1}{m}$$, where m = Rank of T_0. The test statistic

is distributed as F with n - p+1 and m degrees of freedom and the noncentrality paramete
is:

$$\theta = \tfrac{1}{2}\left[T_0\underline{\beta} - \underline{d}_0\right]' \left[A_0A_0' - A_0A'(AA')^{-1}AA_0'\right] \left[T_0\underline{\beta} - \underline{d}_0\right]$$

$$A = T\left[X'X\right]^{-1} X'$$

$$A_0 = T_0\left[X'X\right]^{-1} X'$$

The explicit form of the least square estimates for the P.L.M. is:

$$\tilde{a}_1 = \hat{a}_1 - \frac{\lambda}{2\, m_1}$$

$$\tilde{b}_1 = \hat{b}_1 - \frac{\lambda}{2} \frac{(x^* - \bar{x}_1)}{S_{xx1}}$$

$$\tilde{a}_2 = \hat{a}_2 + \frac{\lambda}{2\, m_2}$$

$$\tilde{b}_2 = \hat{b}_2 + \frac{\lambda}{2} \frac{(x^* - \bar{x}_2)}{S_{xx2}}$$

$$\frac{\lambda}{2} = \frac{\hat{a}_1 + \hat{b}_1(x^* - \bar{x}_1) - \hat{a}_2 - \hat{b}_2(x^* - \bar{x}_2)}{\frac{1}{m_1} + \frac{(x^* - \bar{x}_1)^2}{S_{xx1}} + \frac{1}{m_2} + \frac{(x^* - \bar{x}_2)^2}{S_{xx2}}}$$

where $S_{xx1} = \sum_{l=1}^{m_1} y_l (x_l - \bar{x}_1)$

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